The Conjunction Fallacy

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2. Linda is a bank teller and is active in the feminist movement.

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2. Linda is a bank teller and is active in the feminist movement.

Typically a large percentage of people asked say 2 is more probable than 1.

A. Tversky and D. Kahneman. *Extensions versus intuitive reasoning: The conjunction fallacy in probability judgment*. Psychological Review 90 (4): 293 - 315, 1983.

Conjunction Fallacy

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Which is more probable?

1. Linda is a bank teller. B
2. Linda is a bank teller and is active in the feminist movement. $B \wedge F$

$Pr(B \mid E) \geq Pr(B \wedge F \mid E)$

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$Pr(B \mid E) \geq Pr(B \wedge F \mid E)$

But, E **is positively relevant for** $B \wedge F$ **(and less so than to** B**)**

Conjunction Principle

If E is "good evidence" for $P \wedge Q$, then it is "good evidence" for P.

$$\frac{E \rightarrow (P \wedge Q)}{\therefore \quad E \rightarrow P}$$

P	Q	E	$P \wedge Q$	$E \rightarrow (P \wedge Q)$	$E \rightarrow P$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

$$(E \rightarrow (P \wedge Q)) \models (E \rightarrow P)$$

Evidential Support vs. Relevance

1. Is it true that if E evidentially supports $P \wedge Q$, then E evidentially supports P (If $Pr(P \wedge Q \mid E)$ is high, then so is $Pr(P \mid E)$)?

2. Is it true that if E is positively relevant for $P \wedge Q$, then E is positively relevant for P?

Evidential Support vs. Relevance

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   Yes: If $Pr(P \wedge Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact, for all X, Y, Z, $Pr(X \mid Z) \geq Pr(X \wedge Y \mid Z)$

2. Is it true that if E is positively relevant for $P \wedge Q$, then E is positively relevant for P?

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2. Is it true that if E is positively relevant for $P \wedge Q$, then E is positively relevant for P?

   No: E can be positively relevant for $P \wedge Q$ without being positively relevant to P. That is, $Pr(P \wedge Q \mid E) > Pr(P \wedge Q)$ does not necessarily imply that $Pr(P \mid E) > Pr(P)$.

P	Q	E	$P \wedge Q$	$E \rightarrow (P \wedge Q)$	$E \rightarrow P$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

$$E \rightarrow (P \wedge Q) \models E \rightarrow P$$

	P	Q	E	$P \wedge Q$	$E \rightarrow (P \wedge Q)$	$E \rightarrow P$
p_1	T	T	T	T	T	T
p_2	T	T	F	T	T	T
p_3	T	F	T	F	F	T
p_4	T	F	F	F	T	T
p_5	F	T	T	F	F	F
p_6	F	T	F	F	T	T
p_7	F	F	T	F	F	F
p_8	F	F	F	F	T	T

$$Pr(P \mid E) \geq Pr(P \wedge Q \mid E)$$

	P	Q	E	$P \wedge Q$	$E \to (P \wedge Q)$	$E \to P$
$\frac{p_1}{p_1+p_3+p_5+p_7}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{p_3}{p_1+p_3+p_5+p_7}$	T	F	T	F	F	T
0	T	F	F	F	T	T
$\frac{p_5}{p_1+p_3+p_5+p_7}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{p_7}{p_1+p_3+p_5+p_7}$	F	F	T	F	F	F
0	F	F	F	F	T	T

$$Pr(P \mid E) \geq Pr(P \wedge Q \mid E)$$

	P	Q	E	$P \wedge Q$	$E \rightarrow (P \wedge Q)$	$E \rightarrow P$
$\frac{p_1}{p_1+p_3+p_5+p_7}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{p_3}{p_1+p_3+p_5+p_7}$	T	F	T	F	F	T
0	T	F	F	F	T	T
$\frac{p_5}{p_1+p_3+p_5+p_7}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{p_7}{p_1+p_3+p_5+p_7}$	F	F	T	F	F	F
0	F	F	F	F	T	T

$$Pr(P \mid E) = \frac{p_1+p_3}{p_1+p_3+p_5+p_7} \geq \frac{p_1}{p_1+p_3+p_5+p_7} = Pr(P \wedge Q \mid E)$$

B: The card is black.

A: The card is an ace.

S: The card is a spade.

$Pr(A \wedge S \mid B) > Pr(A \wedge S)$, but $Pr(A \mid B) = Pr(A)$.

	A	S	B	$A \wedge S$	$B \rightarrow (A \wedge S)$	$B \rightarrow A$
$\frac{1}{52}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{1}{52}$	T	F	T	F	F	T
$\frac{2}{52}$	T	F	F	F	T	T
$\frac{12}{52}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{12}{52}$	F	F	T	F	F	F
$\frac{24}{52}$	F	F	F	F	T	T

	A	S	B	$A \wedge S$	$B \to (A \wedge S)$	$B \to A$
$\frac{1}{52}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{1}{52}$	T	F	T	F	F	T
$\frac{2}{52}$	T	F	F	F	T	T
$\frac{12}{52}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{12}{52}$	F	F	T	F	F	F
$\frac{24}{52}$	F	F	F	F	T	T

$Pr(A \wedge S \mid B) = \frac{1}{26} > Pr(A \wedge S) = \frac{1}{52}$, but
$Pr(A \mid B) = Pr(A) = \frac{1}{13}$

	A	S	B	$A \wedge S$	$B \rightarrow (A \wedge S)$	$B \rightarrow A$
$\frac{1}{26}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{1}{26}$	T	F	T	F	F	T
0	T	F	F	F	T	T
$\frac{12}{26}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{12}{26}$	F	F	T	F	F	F
0	F	F	F	F	T	T

$$Pr(A \wedge S \mid B) = \frac{1}{26} > Pr(A \wedge S) = \frac{1}{52}, \text{ but}$$
$$Pr(A \mid B) = Pr(A) = \frac{1}{13}$$

Summary

- A (deductively) valid argument: $E \to (P \wedge Q) \models E \to P$
- If E evidentially supports $P \wedge Q$, then E evidentially supports P.

  If $Pr(P \wedge Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact,

  $$Pr(P \mid E) \geq Pr(P \wedge Q \mid E)$$

- However, E may be positively relevant for $P \wedge Q$ without being positively relevant for P:

  $Pr(P \wedge Q \mid E) > Pr(P \wedge Q)$ does not necessarily imply that $Pr(P \mid E) > Pr(P)$.